IN COLLABORATION WITH IVTB

Diploma in Information Technology

Cohort DIP/03/Full Time

Examinations for 2003 – 2004 / Semester 1

MODULE: MATHS FOR DIGITAL SYSTEM

MODULE CODE: BISE012

Duration: 2 Hours

Instructions to Candidates:

1. Answer any three questions.
2. Always start a new question on a fresh page.
3. All questions carry equal marks.
4. Total marks 75.

This question paper contains 4 questions and 5 pages.
Question 1

(a) Solve the recurrence relation

\[ u_{n+2} = 5u_{n+1} - 6u_n, \quad n \geq 0 \]

with

\[ u_0 = 12 \quad \text{and} \quad u_1 = 31. \]

[10 marks]

(b) Use mathematical induction to prove that

\[ 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n}{6} (n + 1)(2n + 1) \]

for all positive integers \( n \).

[8 marks]

(c) Suppose \( f : \mathbb{R} \rightarrow \mathbb{R} \) is defined by

\[ f(x) = \lfloor x \rfloor. \]

Sketch the graph of \( f \) over the interval \(-3 \leq x \leq 3\). Hence, find \( f^{-1}(\{0\}) \) and \( f^{-1}(\{0,1\}) \).

[7 marks]
Question 2

(a) The function $g : \mathbb{R} \rightarrow \mathbb{R}$ is defined as follows

$$g(x) = x^2 + 8x + 15, \quad x < -4$$

Determine whether $g$ is invertible and if it is, find its inverse. [7 marks]

(b) Suppose $h : A \rightarrow \mathbb{R}$ is defined by

$$h(x) = \frac{3x}{x + 4},$$

where $A = \{x \in \mathbb{R} | x \neq -4\}$. Show that $h$ is injective. [7 marks]

(c) Prove that $\overline{A \cup B} = \overline{A} \cap \overline{B}$ for all sets $A$ and $B$. [11 marks]
Question 3

(a) Find an expression for the output of the following circuit. (Do not simplify your answer)

(b) Use the Karnaugh map to simplify the expression

\[ y = \overline{A} \overline{B} \overline{C} + BC + \overline{A}B. \]

[5 marks]

(c) Simplify the following expressions using Boolean algebra.

(i) \[ z = \overline{A}C(\overline{A}BD) + \overline{A}BC \overline{D} + A\overline{B}C \]

(ii) \[ x = (\overline{A} + B)(A + B + D)\overline{D} \]

[12 marks]
Question 4

(a) Perform the following conversions:

(i) $5674_{10}$ to base 2
(ii) $273_{8}$ to base 2
(iii) $524_{10}$ to base 8
(iv) $FF1_{16}$ to base 8
(v) $100101101001$ (BCD) to decimal

[12 marks]

(b) Consider the truth table below:

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(i) Use the Karnaugh map method to find the simplified form of the output $z$.
(ii) Draw a logic circuit that represents the output $z$ obtained above.

[13 marks]